

Frequency probabilities

# Frequency Probability

- Long run proportion
- Repeatable process

# Notebook/data.frame view

	Species	Color
1	virginica	purple
2	setosa	pink
3	versicolor	pink
⋮	⋮	⋮
K	setosa	pink
K+1	versicolor	pink
⋮	⋮	⋮

$$\#(\text{pink}) = \sum_{i=1}^N I(\text{Color}_i = \text{“pink”})$$

$$P(\text{Color} = \text{“pink”}) = \lim_{N \rightarrow \infty} \frac{\#(\text{pink})}{N}$$

```
df <- function(n) {  
  S <- sample(c("setosa", "versicolor", "virginica"), n, replace=TRUE)  
  pc <- .4*(S=="setosa") + .5*(S=="versicolor") + .2  
  C <- c("purple", "pink")[rbinom(n, 1, pc)+1]  
  data.frame(S = S, C = C)  
}
```

```
> set.seed(1)
> A <- df(50)
> A %>% mutate(event = 1*(C=="pink")) %>% pull(event) %>% mean
[1] 0.44
> A <- rbind(A,df(500))
> A %>% mutate(event = 1*(C=="pink")) %>% pull(event) %>% mean
[1] 0.5163636
> A <- rbind(A,df(5000))
> A %>% mutate(event = 1*(C=="pink")) %>% pull(event) %>% mean
[1] 0.5037838
> A <- rbind(A,df(5000000))
> A %>% mutate(event = 1*(C=="pink")) %>% pull(event) %>% mean
[1] 0.4997297
```

# Joint events

- AND: events created by combining outcomes from two or more features with the AND operator

$$\#(\text{setosa} \ \& \ \text{pink}) = \sum_{i=1}^N I(\text{Species}_i = \text{“setosa”} \ \& \ \text{Color}_i = \text{“pink”})$$

$$P(\text{Species} = \text{“setosa”} \ \& \ \text{Color} = \text{“pink”}) = \lim_{N \rightarrow \infty} \frac{\#(\text{setosa} \ \& \ \text{pink})}{N}$$

# Joint events

- AND: event where two features v

Other symbols for AND depending on the textbook

$\cap, \wedge, \&, \cdot$

$A \cap B, A \wedge B, A \& B, A \cdot B$

$\#(\text{setosa} \&$

more

$i = \text{“pink”})$

$$P(\text{Species} = \text{“setosa”} \& \text{Color} = \text{“pink”}) = \lim_{N \rightarrow \infty} \frac{\#(\text{setosa} \& \text{pink})}{N}$$

# Joint events

- OR: events created by combining outcomes from two or more features with the OR operator

$$\#(\text{setosa} \cup \text{pink}) = \sum_{i=1}^N I(\text{Species}_i = \text{“setosa”} \cup \text{Color}_i = \text{“pink”})$$

$$P(\text{Species} = \text{“setosa”} \cup \text{Color} = \text{“pink”}) = \lim_{N \rightarrow \infty} \frac{\#(\text{setosa} \cup \text{pink})}{N}$$

# Joint events

- OR: event features v

Other symbols for OR depending on the textbook

$\cup, \vee, +, |$

$\#(\text{setosa} \cup$

$\text{pink})$

$A \cup B, A \vee B, A + B, A|B$

$$P(\text{Species} = \text{"setosa"} \cup \text{Color} = \text{"pink"}) = \lim_{N \rightarrow \infty} \frac{\#(\text{setosa} \cup \text{pink})}{N}$$

```
set.seed(1)
df(500) %>%
  mutate(event = 1*(S=="setosa" & C == "pink")) %>%
  pull(event) %>%
  mean
```

```
df(500) %>%
  mutate(event = 1*(S=="setosa" | C == "pink")) %>%
  pull(event) %>%
  mean
```

Cross Tab

# Pet Color Example

## Cell frequencies

pet	blue	green	red	Row Total
cat	52	269	73	394
dog	299	290	17	606
Column Total	351	559	90	1000

# Pet Color Example

## Cell proportions

pet	blue	green	red	Row Total
cat	$\frac{52}{1000}$	$\frac{269}{1000}$	$\frac{73}{1000}$	$\frac{394}{1000}$
dog	$\frac{299}{1000}$	$\frac{290}{1000}$	$\frac{17}{1000}$	$\frac{606}{1000}$
Column Total	$\frac{351}{1000}$	$\frac{559}{1000}$	$\frac{90}{1000}$	$\frac{1000}{1000}$

# Pet Color Example

N = 1000

pet	blue	green	red	Row Total
cat	0.052	0.269	0.073	0.394
dog	0.299	0.290	0.017	0.606
Column Total	0.351	0.599	0.090	1

Cell frequency

Cell proportion

pet	blue	green	red	Row Total
cat	52	269	73	394
	0.052	0.269	0.073	0.394
dog	299	290	17	606
	0.299	0.290	0.017	0.606
Column Total	351	559	90	1000
	0.351	0.599	0.090	1

pet	blue	green	red	Row Total
Cell frequency	52	269	73	394
Cell proportion	0.052	0.269	0.073	0.394
Row proportion	$\frac{52}{394}$	$\frac{269}{394}$	$\frac{73}{394}$	
dog	299	290	17	606
	0.299	0.290	0.017	0.606
	$\frac{299}{606}$	$\frac{290}{606}$	$\frac{17}{606}$	
Column Total	351	559	90	1000
	0.351	0.599	0.090	1

	pet	blue	green	red	Row Total
Cell frequency		52	269	73	394
Cell proportion		0.052	0.269	0.073	0.394
Row proportion		0.132	0.683	0.185	
	dog	299	290	17	606
		0.299	0.290	0.017	0.606
		0.493	0.479	0.028	
	Column Total	351	559	90	1000
		0.351	0.599	0.090	1

What is the sum of row proportions?

pet	blue	green	red	Row Total
cat	52	269	73	394
	0.052	0.269	0.073	0.394
	0.132	0.683	0.185	
dog	299	290	17	606
	0.299	0.290	0.017	0.606
	0.493	0.479	0.028	
Column Total	351	559	90	1000
	0.351	0.599	0.090	1

	pet	blue	green	red	Row Total
Cell frequency		52	269	73	394
Cell proportion		0.052	0.269	0.073	0.394
Row proportion		0.132	0.683	0.185	
Column proportion		$\frac{52}{351}$	$\frac{269}{559}$	$\frac{73}{90}$	
	dog	299	290	17	606
		0.299	0.290	0.017	0.606
		0.493	0.479	0.028	
		$\frac{299}{351}$	$\frac{290}{559}$	$\frac{17}{90}$	
	Column Total	351	559	90	1000

pet	blue	green	red	Row Total
cat	52	269	73	394
	0.052	0.269	0.073	0.394
	0.132	0.683	0.185	
Sum?	$\frac{52}{351}$	$\frac{269}{559}$	$\frac{73}{90}$	
dog	299	290	17	606
	0.299	0.290	0.017	0.606
	0.493	0.479	0.028	
	$\frac{299}{351}$	$\frac{290}{559}$	$\frac{17}{90}$	
Column Total	351	559	90	1000

Column proportions  
sum to 1

pet	blue	green	red	Row Total
cat	52	269	73	394
	0.052	0.269	0.073	0.394
	0.132	0.683	0.185	
	$\frac{52}{351}$	$\frac{269}{559}$	$\frac{73}{90}$	
dog	299	290	17	606
	0.299	0.290	0.017	0.606
	0.493	0.479	0.028	
	$\frac{299}{351}$	$\frac{290}{559}$	$\frac{17}{90}$	
Column Total	351	559	90	1000

pet	blue	green	red	Row Total
cat	52	269	73	394
Cell proportion	0.052	0.269	0.073	0.394
Row proportion	0.132	0.683	0.185	
Col proportion	0.148	0.481	0.811	
dog	299	290	17	606
	0.299	0.290	0.017	0.606
	0.493	0.479	0.028	
	0.852	0.519	0.189	
Column Total	351	559	90	1000
	0.351	0.599	0.090	1

```
# Many different ways to  
# generate cross tabs in R  
  
xtabs(~pet+color, data = df1)  
  
table(df1$pet, df1$color)  
  
gmodels::CrossTable(  
  df1$pet  
  , df1$color  
  , prop.chisq = FALSE  
)
```

Cell Contents

-----
N
N / Row Total
N / Col Total
N / Table Total
-----

Total Observations in Table: 1000

	color			
pet	blue	green	red	Row Total
cat	52	269	73	394
	0.132	0.683	0.185	0.394
	0.148	0.481	0.811	
	0.052	0.269	0.073	
dog	299	290	17	606
	0.493	0.479	0.028	0.606
	0.852	0.519	0.189	
	0.299	0.290	0.017	
Column Total	351	559	90	1000
	0.351	0.559	0.090	

Cell Contents

	N
N / Row Total	
N / Col Total	
N / Table Total	

Total Observations in Table: 100000

pet	color			Row Total
	blue	green	red	
cat	6308	27277	6449	40034
	0.158	0.681	0.161	0.400
	0.173	0.489	0.826	
	0.063	0.273	0.064	
dog	30052	28555	1359	59966
	0.501	0.476	0.023	0.600
	0.827	0.511	0.174	
	0.301	0.286	0.014	
Column Total	36360	55832	7808	100000
	0.364	0.558	0.078	

Cell Contents

-----	
	N
N / Row Total	
N / Col Total	
N / Table Total	
-----	

Total Observations in Table: 10000000

pet	color			Row Total
	blue	green	red	
cat	635702	2728977	634997	3999676
	0.159	0.682	0.159	0.400
	0.175	0.488	0.823	
	0.064	0.273	0.063	
dog	2998691	2865233	136400	6000324
	0.500	0.478	0.023	0.600
	0.825	0.512	0.177	
	0.300	0.287	0.014	
Column Total	3634393	5594210	771397	10000000
	0.363	0.559	0.077	

# Pet Color Example

limit as  $N \rightarrow \infty$

pet	blue	green	red	Row Total
cat	0.0634621	0.2730758	0.0634621	0.4
dog	0.3	0.2863499	0.01365008	0.6
Column Total	0.3634621	0.5594257	0.07711218	1

limit as  $N \rightarrow \infty$

pet	blue	green	red	Row Total
cat	$\infty$	$\infty$	$\infty$	$\infty$
Cell prob	$P(\text{cat \& blue})$	$P(\text{cat \& green})$	$P(\text{cat \& red})$	$P(\text{cat})$
Row prob	$P(\text{blue} \mid \text{cat})$	$P(\text{green} \mid \text{cat})$	$P(\text{red} \mid \text{cat})$	
Col prob	$P(\text{cat} \mid \text{blue})$	$P(\text{cat} \mid \text{green})$	$P(\text{cat} \mid \text{red})$	
dog	$\infty$	$\infty$	$\infty$	$\infty$
Cell prob	$P(\text{dog \& blue})$	$P(\text{dog \& green})$	$P(\text{dog \& red})$	$P(\text{dog})$
Row prob	$P(\text{blue} \mid \text{dog})$	$P(\text{green} \mid \text{dog})$	$P(\text{red} \mid \text{dog})$	
Col prob	$P(\text{dog} \mid \text{blue})$	$P(\text{dog} \mid \text{green})$	$P(\text{dog} \mid \text{red})$	
Column Total	$\infty$	$\infty$	$\infty$	$\infty$
	P(blue)	P(green)	P(red)	1

limit as  $N \rightarrow \infty$

pet	blue	green	red	Row Total
cat				
Cell prob	$P(\text{cat \& blue})$	$P(\text{cat \& green})$	$P(\text{cat \& red})$	$P(\text{cat})$
Row prob	$P(\text{blue} \mid \text{cat})$	$P(\text{green} \mid \text{cat})$	$P(\text{red} \mid \text{cat})$	
Col prob	$P(\text{cat} \mid \text{blue})$	$P(\text{cat} \mid \text{green})$	$P(\text{cat} \mid \text{red})$	
dog				
Cell prob	$P(\text{dog \& blue})$	$P(\text{dog \& green})$	$P(\text{dog \& red})$	$P(\text{dog})$
Row prob	$P(\text{blue} \mid \text{dog})$	$P(\text{green} \mid \text{dog})$	$P(\text{red} \mid \text{dog})$	
Col prob	$P(\text{dog} \mid \text{blue})$	$P(\text{dog} \mid \text{green})$	$P(\text{dog} \mid \text{red})$	
Column Total	$P(\text{blue})$	$P(\text{green})$	$P(\text{red})$	1

Marginal  
Probability



limit as  $N \rightarrow \infty$

pet		blue	green	red	Row Total
cat	Cell prob	$P(\text{cat \& blue})$	$P(\text{cat \& green})$	$P(\text{cat \& red})$	$P(\text{cat})$
	Row prob	$P(\text{blue} \mid \text{cat})$	$P(\text{green} \mid \text{cat})$	$P(\text{red} \mid \text{cat})$	
	Col prob	$P(\text{cat} \mid \text{blue})$	$P(\text{cat} \mid \text{green})$	$P(\text{cat} \mid \text{red})$	
dog	Cell prob	$P(\text{dog \& blue})$	$P(\text{dog \& green})$	$P(\text{dog \& red})$	$P(\text{dog})$
	Row prob	$P(\text{blue} \mid \text{dog})$	$P(\text{green} \mid \text{dog})$	$P(\text{red} \mid \text{dog})$	
	Col prob	$P(\text{dog} \mid \text{blue})$	$P(\text{dog} \mid \text{green})$	$P(\text{dog} \mid \text{red})$	
Column Total		$P(\text{blue})$	$P(\text{green})$	$P(\text{red})$	1

Marginal  
Probability



limit as  $N \rightarrow \infty$

pet		blue	green	red	Row Total
cat	Cell prob	$P(\text{cat \& blue})$	$P(\text{cat \& green})$	$P(\text{cat \& red})$	$P(\text{cat})$
	Row prob	$P(\text{blue} \mid \text{cat})$	$P(\text{green} \mid \text{cat})$	$P(\text{red} \mid \text{cat})$	
	Col prob	$P(\text{cat} \mid \text{blue})$	$P(\text{cat} \mid \text{green})$	$P(\text{cat} \mid \text{red})$	
dog	Cell prob	$P(\text{dog \& blue})$	$P(\text{dog \& green})$	$P(\text{dog \& red})$	$P(\text{dog})$
	Row prob	$P(\text{blue} \mid \text{dog})$	$P(\text{green} \mid \text{dog})$	$P(\text{red} \mid \text{dog})$	
	Col prob	$P(\text{dog} \mid \text{blue})$	$P(\text{dog} \mid \text{green})$	$P(\text{dog} \mid \text{red})$	
Column Total		$P(\text{blue})$	$P(\text{green})$	$P(\text{red})$	1

Cell Probability  
Joint Probability

Marginal  
Probability

limit as  $N \rightarrow \infty$

pet	blue	green	red	Row Total	
cat	Cell prob	$P(\text{cat \& blue})$	$P(\text{cat \& green})$	$P(\text{cat \& red})$	$P(\text{cat})$
	Row prob	$P(\text{blue} \mid \text{cat})$	$P(\text{green} \mid \text{cat})$	$P(\text{red} \mid \text{cat})$	
	Col prob	$P(\text{cat} \mid \text{blue})$	$P(\text{cat} \mid \text{green})$	$P(\text{cat} \mid \text{red})$	
dog	Cell prob	$P(\text{dog \& blue})$	$P(\text{dog \& green})$	$P(\text{dog \& red})$	$P(\text{dog})$
	Row prob	$P(\text{blue} \mid \text{dog})$	$P(\text{green} \mid \text{dog})$	$P(\text{red} \mid \text{dog})$	
	Col prob	$P(\text{dog} \mid \text{blue})$	$P(\text{dog} \mid \text{green})$	$P(\text{dog} \mid \text{red})$	
Column Total	$P(\text{blue})$	$P(\text{green})$	$P(\text{red})$	1	

Cell Probability  
Joint Probability

Marginal  
Probability

limit as  $N \rightarrow \infty$

pet	blue	green	red	Row Total	
cat	Cell prob	$P(\text{cat \& blue})$	$P(\text{cat \& green})$	$P(\text{cat \& red})$	$P(\text{cat})$
	Row prob	$P(\text{blue} \mid \text{cat})$	$P(\text{green} \mid \text{cat})$	$P(\text{red} \mid \text{cat})$	
	Col prob	$P(\text{cat} \mid \text{blue})$	$P(\text{cat} \mid \text{green})$	$P(\text{cat} \mid \text{red})$	
dog	Cell prob	$P(\text{dog \& blue})$	$P(\text{dog \& green})$	$P(\text{dog \& red})$	$P(\text{dog})$
	Row prob	$P(\text{blue} \mid \text{dog})$	$P(\text{green} \mid \text{dog})$	$P(\text{red} \mid \text{dog})$	
	Col prob	$P(\text{dog} \mid \text{blue})$	$P(\text{dog} \mid \text{green})$	$P(\text{dog} \mid \text{red})$	
Column Total	$P(\text{blue})$	$P(\text{green})$	$P(\text{red})$	1	

Conditional Probabilities

pet	blue	green	red	Row Total	
cat	Cell prob	$P(\text{cat \& blue})$	$P(\text{cat \& green})$	$P(\text{cat \& red})$	$P(\text{cat})$
	Row prob	$P(\text{blue} \mid \text{cat})$	$P(\text{green} \mid \text{cat})$	$P(\text{red} \mid \text{cat})$	
	Col prob	$P(\text{cat} \mid \text{blue})$	$P(\text{cat} \mid \text{green})$	$P(\text{cat} \mid \text{red})$	
dog	Cell prob	$P(\text{dog \& blue})$	$P(\text{dog \& green})$	$P(\text{dog \& red})$	$P(\text{dog})$
	Row prob	$P(\text{blue} \mid \text{dog})$	$P(\text{green} \mid \text{dog})$	$P(\text{red} \mid \text{dog})$	
	Col prob	$P(\text{dog} \mid \text{blue})$	$P(\text{dog} \mid \text{green})$	$P(\text{dog} \mid \text{red})$	
Column Total	$P(\text{blue})$	$P(\text{green})$	$P(\text{red})$	1	

**RULE:** All probabilities are between 0 and 1

$$0 \leq P \leq 1$$

pet	blue	green	red	Row Total	
cat	Cell prob	$P(\text{cat \& blue})$	$P(\text{cat \& green})$	$P(\text{cat \& red})$	$P(\text{cat})$
	Row prob	$P(\text{blue} \mid \text{cat})$	$P(\text{green} \mid \text{cat})$	$P(\text{red} \mid \text{cat})$	
	Col prob	$P(\text{cat} \mid \text{blue})$	$P(\text{cat} \mid \text{green})$	$P(\text{cat} \mid \text{red})$	
dog	Cell prob	$P(\text{dog \& blue})$	$P(\text{dog \& green})$	$P(\text{dog \& red})$	$P(\text{dog})$
	Row prob	$P(\text{blue} \mid \text{dog})$	$P(\text{green} \mid \text{dog})$	$P(\text{red} \mid \text{dog})$	
	Col prob	$P(\text{dog} \mid \text{blue})$	$P(\text{dog} \mid \text{green})$	$P(\text{dog} \mid \text{red})$	
Column Total	$P(\text{blue})$	$P(\text{green})$	$P(\text{red})$	1	

**RULE:** Probabilities of all possible outcomes sum to 1.

$$\sum_{i=1}^J P_i = 1$$

pet	blue	green	red	Row Total	
cat	Cell prob	$P(\text{cat \& blue})$	$P(\text{cat \& green})$	$P(\text{cat \& red})$	$P(\text{cat})$
	Row prob	$P(\text{blue} \mid \text{cat})$	$P(\text{green} \mid \text{cat})$	$P(\text{red} \mid \text{cat})$	
	Col prob	$P(\text{cat} \mid \text{blue})$	$P(\text{cat} \mid \text{green})$	$P(\text{cat} \mid \text{red})$	
dog	Cell prob	$P(\text{dog \& blue})$	$P(\text{dog \& green})$	$P(\text{dog \& red})$	$P(\text{dog})$
	Row prob	$P(\text{blue} \mid \text{dog})$	$P(\text{green} \mid \text{dog})$	$P(\text{red} \mid \text{dog})$	
	Col prob	$P(\text{dog} \mid \text{blue})$	$P(\text{dog} \mid \text{green})$	$P(\text{dog} \mid \text{red})$	
Column Total	$P(\text{blue})$	$P(\text{green})$	$P(\text{red})$	1	

$$\begin{aligned}
 &P(\text{cat \& blue}) + P(\text{cat \& green}) + P(\text{cat \& red}) \\
 &+ P(\text{dog \& blue}) + P(\text{dog \& green}) + P(\text{dog \& red}) = 1
 \end{aligned}$$

pet	blue	green	red	Row Total	
cat	Cell prob	$P(\text{cat \& blue})$	$P(\text{cat \& green})$	$P(\text{cat \& red})$	$P(\text{cat})$
	Row prob	$P(\text{blue} \mid \text{cat})$	$P(\text{green} \mid \text{cat})$	$P(\text{red} \mid \text{cat})$	
	Col prob	$P(\text{cat} \mid \text{blue})$	$P(\text{cat} \mid \text{green})$	$P(\text{cat} \mid \text{red})$	
dog	Cell prob	$P(\text{dog \& blue})$	$P(\text{dog \& green})$	$P(\text{dog \& red})$	$P(\text{dog})$
	Row prob	$P(\text{blue} \mid \text{dog})$	$P(\text{green} \mid \text{dog})$	$P(\text{red} \mid \text{dog})$	
	Col prob	$P(\text{dog} \mid \text{blue})$	$P(\text{dog} \mid \text{green})$	$P(\text{dog} \mid \text{red})$	
Column Total	$P(\text{blue})$	$P(\text{green})$	$P(\text{red})$	1	

$$P(\text{cat} \mid \text{blue}) + P(\text{dog} \mid \text{blue}) = 1$$

pet	blue	green	red	Row Total	
cat	Cell prob	$P(\text{cat \& blue})$	$P(\text{cat \& green})$	$P(\text{cat \& red})$	$P(\text{cat})$
	Row prob	$P(\text{blue} \mid \text{cat})$	$P(\text{green} \mid \text{cat})$	$P(\text{red} \mid \text{cat})$	
	Col prob	$P(\text{cat} \mid \text{blue})$	$P(\text{cat} \mid \text{green})$	$P(\text{cat} \mid \text{red})$	
dog	Cell prob	$P(\text{dog \& blue})$	$P(\text{dog \& green})$	$P(\text{dog \& red})$	$P(\text{dog})$
	Row prob	$P(\text{blue} \mid \text{dog})$	$P(\text{green} \mid \text{dog})$	$P(\text{red} \mid \text{dog})$	
	Col prob	$P(\text{dog} \mid \text{blue})$	$P(\text{dog} \mid \text{green})$	$P(\text{dog} \mid \text{red})$	
Column Total	$P(\text{blue})$	$P(\text{green})$	$P(\text{red})$	1	

$$P(\text{blue} \mid \text{dog}) + P(\text{green} \mid \text{dog}) + P(\text{red} \mid \text{dog}) = 1$$

pet	blue	green	red	Row Total	
cat	Cell prob	$P(\text{cat \& blue})$	$P(\text{cat \& green})$	$P(\text{cat \& red})$	$P(\text{cat})$
	Row prob	$P(\text{blue} \mid \text{cat})$	$P(\text{green} \mid \text{cat})$	$P(\text{red} \mid \text{cat})$	
	Col prob	$P(\text{cat} \mid \text{blue})$	$P(\text{cat} \mid \text{green})$	$P(\text{cat} \mid \text{red})$	
dog	Cell prob	$P(\text{dog \& blue})$	$P(\text{dog \& green})$	$P(\text{dog \& red})$	$P(\text{dog})$
	Row prob	$P(\text{blue} \mid \text{dog})$	$P(\text{green} \mid \text{dog})$	$P(\text{red} \mid \text{dog})$	
	Col prob	$P(\text{dog} \mid \text{blue})$	$P(\text{dog} \mid \text{green})$	$P(\text{dog} \mid \text{red})$	
Column Total	$P(\text{blue})$	$P(\text{green})$	$P(\text{red})$	1	

$$P(\text{cat}) + P(\text{dog}) = 1$$

pet	blue	green	red	Row Total
cat				
Cell prob	$P(\text{cat \& blue})$	$P(\text{cat \& green})$	$P(\text{cat \& red})$	$P(\text{cat})$
Row prob	$P(\text{blue} \mid \text{cat})$	$P(\text{green} \mid \text{cat})$	$P(\text{red} \mid \text{cat})$	
Col prob	$P(\text{cat} \mid \text{blue})$	$P(\text{cat} \mid \text{green})$	$P(\text{cat} \mid \text{red})$	
dog				
Cell prob	$P(\text{dog \& blue})$	$P(\text{dog \& green})$	$P(\text{dog \& red})$	$P(\text{dog})$
Row prob	$P(\text{blue} \mid \text{dog})$	$P(\text{green} \mid \text{dog})$	$P(\text{red} \mid \text{dog})$	
Col prob	$P(\text{dog} \mid \text{blue})$	$P(\text{dog} \mid \text{green})$	$P(\text{dog} \mid \text{red})$	
Column Total	$P(\text{blue})$	$P(\text{green})$	$P(\text{red})$	1

**Law of total probability:** Cell probabilities on the same row sum to the marginal probability.

$$P(\text{dog \& blue}) + P(\text{dog \& green}) + P(\text{dog \& red}) = P(\text{dog})$$

pet		blue	green	red	Row Total
cat	Cell prob	$P(\text{cat \& blue})$	$P(\text{cat \& green})$	$P(\text{cat \& red})$	$P(\text{cat})$
	Row prob	$P(\text{blue} \mid \text{cat})$	$P(\text{green} \mid \text{cat})$	$P(\text{red} \mid \text{cat})$	
	Col prob	$P(\text{cat} \mid \text{blue})$	$P(\text{cat} \mid \text{green})$	$P(\text{cat} \mid \text{red})$	
dog	Cell prob	$P(\text{dog \& blue})$	$P(\text{dog \& green})$	$P(\text{dog \& red})$	$P(\text{dog})$
	Row prob	$P(\text{blue} \mid \text{dog})$	$P(\text{green} \mid \text{dog})$	$P(\text{red} \mid \text{dog})$	
	Col prob	$P(\text{dog} \mid \text{blue})$	$P(\text{dog} \mid \text{green})$	$P(\text{dog} \mid \text{red})$	
Column Total		$P(\text{blue})$	$P(\text{green})$	$P(\text{red})$	1

**Law of total probability:** Cell probabilities in the same column sum to the marginal probability.

$$P(\text{cat \& green}) + P(\text{dog \& green}) = P(\text{green})$$

pet	blue	green	red	Row Total
cat				
Cell prob	$P(\text{cat \& blue})$	$P(\text{cat \& green})$	$P(\text{cat \& red})$	???
Row prob	???	???	???	
Col prob	???	???	???	
dog				
Cell prob	$P(\text{dog \& blue})$	???	$P(\text{dog \& red})$	???
Row prob	???	???	???	
Col prob	???	???	???	
Column Total	???	???	???	???

**Question: Is there enough information to fill in the rest of the table?**

pet	blue	green	red	Row Total
cat				
Cell prob	0.2	0.1	0.1	???
Row prob	???	???	???	
Col prob	???	???	???	
dog				
Cell prob	0.1	???	0.3	???
Row prob	???	???	???	
Col prob	???	???	???	
Column Total	???	???	???	???

Question: Is there enough information to fill in the rest of the table?

pet	blue	green	red	Row Total
cat				
Cell prob	0.2	0.1	0.1	???
Row prob	???	???	???	
Col prob	???	???	???	
dog				
Cell prob	0.1	???	0.3	???
Row prob	???	???	???	
Col prob	???	???	???	
Column Total	???	???	???	1

pet	blue	green	red	Row Total
cat				
Cell prob	0.2	0.1	0.1	???
Row prob	???	???	???	
Col prob	???	???	???	
dog				
Cell prob	0.1	0.2	0.3	???
Row prob	???	???	???	
Col prob	???	???	???	
Column Total	???	???	???	1

pet	blue	green	red	Row Total
cat				
Cell prob	0.2	0.1	0.1	$0.2+0.1+0.1=0.4$
Row prob	???	???	???	
Col prob	???	???	???	
dog				
Cell prob	0.1	0.2	0.3	$0.1+0.2+0.3 = 0.6$
Row prob	???	???	???	
Col prob	???	???	???	
Column Total	???	???	???	1

pet	blue	green	red	Row Total
cat				
Cell prob	0.2	0.1	0.1	0.4
Row prob	???	???	???	
Col prob	???	???	???	
dog				
Cell prob	0.1	0.2	0.3	0.6
Row prob	???	???	???	
Col prob	???	???	???	
Column Total	???	???	???	1

pet	blue	green	red	Row Total
cat				
Cell prob	0.2	0.1	0.1	0.4
Row prob	$\frac{0.2}{0.4}$	$\frac{0.1}{0.4}$	$\frac{0.1}{0.4}$	
Col prob	???	???	???	
dog				
Cell prob	0.1	0.2	0.3	0.6
Row prob	???	???	???	
Col prob	???	???	???	
Column Total	???	???	???	1

pet	blue	green	red	Row Total
cat				
Cell prob	0.2	0.1	0.1	0.4
Row prob	$\frac{0.2}{0.4}$	$\frac{0.1}{0.4}$	$\frac{0.1}{0.4}$	
Col prob	???	???	???	
dog				
Cell prob	0.1	0.2	0.3	0.6
Row prob	$\frac{0.1}{0.6}$	$\frac{0.2}{0.6}$	$\frac{0.3}{0.6}$	
Col prob	???	???	???	
Column Total	???	???	???	1

pet		blue	green	red	Row Total
cat	Cell prob	0.2	0.1	0.1	0.4
	Row prob	$\frac{0.2}{0.4}$	$\frac{0.1}{0.4}$	$\frac{0.1}{0.4}$	
	Col prob	???	???	???	
dog	Cell prob	0.1	0.2	0.3	0.6
	Row prob	$\frac{0.1}{0.6}$	$\frac{0.2}{0.6}$	$\frac{0.3}{0.6}$	
	Col prob	???	???	???	
Column Total		$0.2+0.1 = 0.3$	$0.1+0.2=0.3$	$0.1+0.3=0.4$	1

pet		blue	green	red	Row Total
cat	Cell prob	0.2	0.1	0.1	0.4
	Row prob	$\frac{0.2}{0.4}$	$\frac{0.1}{0.4}$	$\frac{0.1}{0.4}$	
	Col prob	???	???	???	
dog	Cell prob	0.1	0.2	0.3	0.6
	Row prob	$\frac{0.1}{0.6}$	$\frac{0.2}{0.6}$	$\frac{0.3}{0.6}$	
	Col prob	???	???	???	
Column Total		<b>0.3</b>	<b>0.3</b>	<b>0.4</b>	1

pet		blue	green	red	Row Total
cat	Cell prob	0.2	0.1	0.1	0.4
	Row prob	$\frac{0.2}{0.4}$	$\frac{0.1}{0.4}$	$\frac{0.1}{0.4}$	
	Col prob	$\frac{0.2}{0.3}$		???	
dog	Cell prob	0.1	0.2	0.3	0.6
	Row prob	$\frac{0.1}{0.6}$	$\frac{0.2}{0.6}$	$\frac{0.3}{0.6}$	
	Col prob	$\frac{0.1}{0.3}$	???	???	
Column Total		0.3	0.3	0.4	1

pet		blue	green	red	Row Total
cat	Cell prob	0.2	0.1	0.1	0.4
	Row prob	$\frac{0.2}{0.4}$	$\frac{0.1}{0.4}$	$\frac{0.1}{0.4}$	
	Col prob	$\frac{0.2}{0.3}$	$\frac{0.1}{0.3}$	???	
dog	Cell prob	0.1	0.2	0.3	0.6
	Row prob	$\frac{0.1}{0.6}$	$\frac{0.2}{0.6}$	$\frac{0.3}{0.6}$	
	Col prob	$\frac{0.1}{0.3}$	$\frac{0.2}{0.3}$	???	
Column Total		0.3	0.3	0.4	1

pet		blue	green	red	Row Total
cat	Cell prob	0.2	0.1	0.1	0.4
	Row prob	$\frac{0.2}{0.4}$	$\frac{0.1}{0.4}$	$\frac{0.1}{0.4}$	
	Col prob	$\frac{0.2}{0.3}$	$\frac{0.1}{0.3}$	$\frac{0.1}{0.4}$	
dog	Cell prob	0.1	0.2	0.3	0.6
	Row prob	$\frac{0.1}{0.6}$	$\frac{0.2}{0.6}$	$\frac{0.3}{0.6}$	
	Col prob	$\frac{0.1}{0.3}$	$\frac{0.2}{0.3}$	$\frac{0.1}{0.4}$	
Column Total		0.3	0.3	0.4	1

pet	blue	green	red	Row Total
cat				
Cell prob	???	???	???	0.3
Row prob	0.2	0.5	???	
Col prob	???	???	???	
dog				
Cell prob	???	???	???	???
Row prob	0.3	???	0.6	
Col prob	???	???	???	
Column Total	???	???	???	???

**Question: Is there enough information to fill in the rest of the table?**